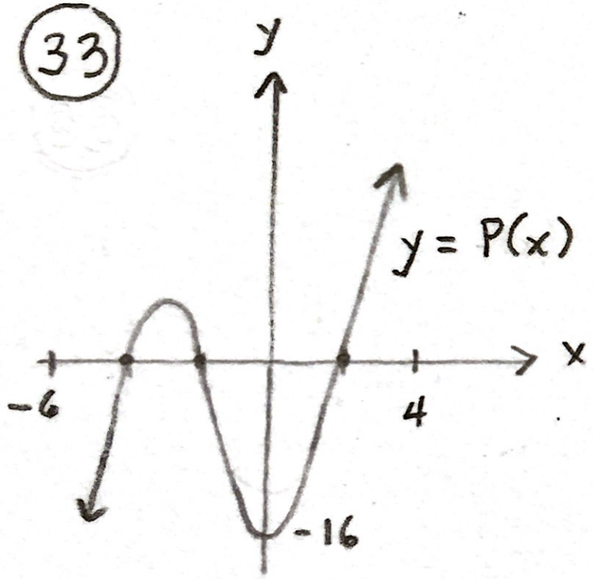
4.7 – Solving Polynomial / Rational Inequalities
(Graphical vs. Symbolic Methods)

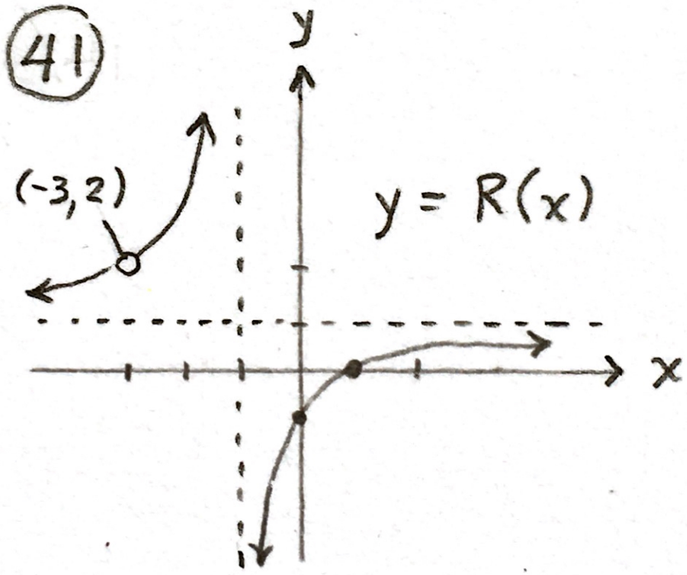
Graphical Method

Examples. Solve for x: (33) $P(x) > 0$, (41) $R(x) \geq 0$

(33)



(41)



Symbolic Method

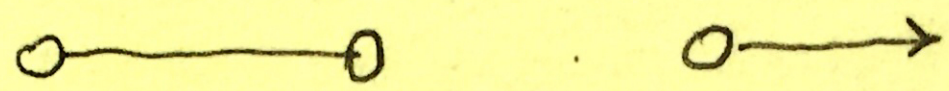
(33) $x^3 + 4x^2 - 4x - 16 > 0$

(41) $\dfrac{x^2 + 2x - 3}{x^2 + 4x + 3} \geq 0$

4.7 – Symbolic Method (cont'd)

(33) $P(x) > 0$

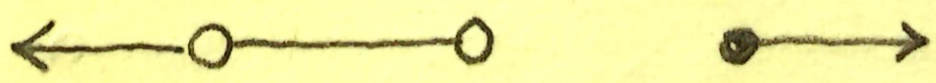
x	-5	-4	-3	-2	0	2	3
x+4	-		+		+		+
x+2	-		-		+		+
x-2	-		-		-		+
P(x)	-	0	+	0	-	0	+



ANS: $(-4,-2) \cup (2,\infty)$

(41) $R(x) \geq 0$

x	-4	-3	-2	-1	0	1	2
x-1	-		-		-		+
x+1	-		-		+		+
R(x)	+	U	+	U	-	0	+



ANS: $(-\infty,-3) \cup (-3,-1) \cup [1,\infty)$

4.7 – Symbolic Method (cont'd)

Procedures

Polynomial Inequalities

1. Isolate the function.
2. Solve $P(x) = 0$ to find the boundary numbers.
3. Use the boundary numbers to partition the number line and pick test values.
4. Set up table to determine where $P(x)$ is +/-.

Rational Inequalities $\left(R(x) = \frac{p(x)}{q(x)}\right)$

2. Solve $p(x) = 0$ and $q(x) = 0$; otherwise same as above.

4.8 – Radical Equations

Examples. $x^{2/3} - x^{1/3} - 6 = 0$

$2n^{-2} - n^{-1} - 3 = 0$

Prep. 1. Simplifying (by hand): $8^{2/3}$, $16^{3/4}$ [use $a^{n/m} = (a^{1/m})^n$]

2. Conversions: radicals $\longleftrightarrow$ fractional exponents

Example. $\sqrt[3]{x^5} = x^{5/3}$

3. [use $a^{n/m} = \sqrt[m]{a^n}$] (SEE EXAMPLES)

3. Reducible Quadratic Equations

Example. $x^4 - 5x^2 + 4 = 0$

(let $y = x^2$)

[in general, use y = Middle term w/o coeff]

4.8 (cont'd) – Conversion Examples

(22) $\sqrt[5]{x^2}$

(23) $(\sqrt[4]{y})^{-3}$

(25) $\sqrt{x} \cdot \sqrt[3]{x}$

(27) $\sqrt{y \cdot \sqrt{y}}$ (called a "nested" radical)

4.8 (contid) - Solving Rad Equations

Review. $x^2 = 4$

$x^3 = 8$

$x^{1/2} = 3$

$x^{1/3} = 3$

$x^{3/2} = 8$

$x^{2/3} = 9$

If $a^{n/m} = b$, then:

$$\begin{cases} a = b^{m/n} & \text{if } n\text{-odd} \\ a = \pm b^{m/n} & \text{if } n\text{-even} \end{cases}$$

(115) $x^{4/3} = 16$

⊙ $(x+3)^{4/3} = 16$

(117) $4x^{3/2} + 5 = 21$

(123) $x^{2/3} - x^{1/3} - 6 = 0$ (Then try 6/5, 3/5.)

⊙ $x^{4/3} - 5x^{2/3} + 4 = 0$

(120)* $2n^{-2} - n^{-1} - 3 = 0$

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* This is a Rational Equation.